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Evaluation of the momentum equation for a turbulent wall jet

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Experimental evaluation of the radial momentum equation near the surface for an axisymmetric turbulent wall jet is reported. The equation for flow contains six terms of magnitudes which cannot be neglected. The pressure gradient across the flow as well as along the flow is found to be of major importance. Thus, for a wall jet flow it is impossible to treat the radial and vertical momentum equations as independent of one another.

1. Introduction

The flow outward from an axisymmetric air jet impinging on a solid surface is of importance in many current applications. For example, vertical take-off aircraft and ground-effect vehicles produce this particular type of flow. Jets impinging on solid surfaces are also of importance in hydraulic applications. Thus, there is a need to understand and predict the features of this particular type of flow.

Glauert (1956) has obtained an empirical expression for the mean velocity distribution of wall jets and his predictions are found to be reasonably good for both rough and smooth walls. There is, however, no detailed information on the wall jet.

The present paper contains a set of experimental measurements sufficient to evaluate the radial equation of motion. As in all turbulent shear flows, it is necessary to resort to experimental measurements in order to establish the applicable equations of motion. The radial equation of motion for the wall jet appears to be more complicated than that encountered in most turbulent shear flows. Evidence is also presented to indicate that the vertical equation of motion is not as simple as that found for boundary-layer flows.

2. Test set-up and procedure

The general arrangement of the test facility is shown in figure 1. A centrifugal pump driven by a 5 h.p. induction motor supplied the air to a chamber through a 5 in. pipe. Two layers of screen with a 0·025 in. square opening were placed in the mixing chamber to assist in attaining uniform flow. The air jet was formed by a sharp-edged orifice of 2 in. diameter. The velocity at the orifice, 2 ft. above the plate surface, was maintained at 340 ft./sec. For the measurements reported herein the surface was covered with lead shot 0·18 in. in diameter. This was packed as closely as possible to give a maximum density in a given area. The arrangement of the shot shows an axisymmetric pattern at 60° intervals with respect to the vertical axis of the impinging jet. The surface plate was 138 in. in diameter, and the jet was located directly above the centre. Measurements of the mean and turbulent velocities at 60° intervals around the plate indicated that the flow was axisymmetrical.

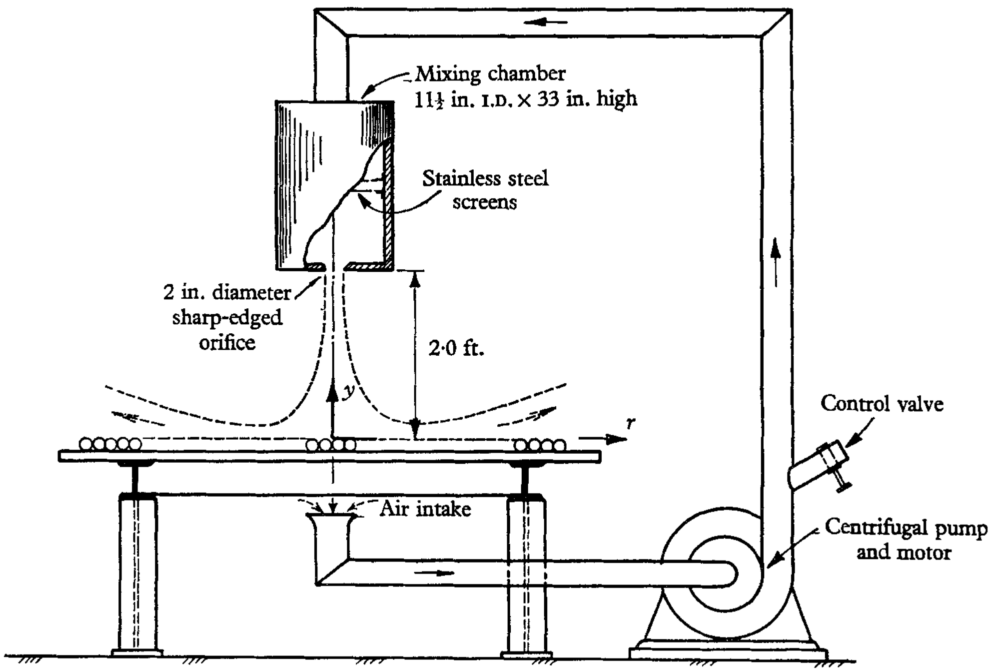

FIGURE 1. General set-up of experiment (not to scale).

The measurements of the wall jet were taken at 28, 30 and 32 in. from the centre of the impinging jet. Tsuei (1962) reported that the turbulent intensities (when the orifice opening was 1 in. and the velocity at the orifice was 370 ft./sec at the jet axis 12 in. below the orifice opening) were approximately 18 % for the radial and circumferential directions and 21 % for the vertical direction. The static-pressure distribution across the flow was measured with a static tube having an outside diameter of 0·062 in. The static tube was alined to the measured mean direction of flow. Tests in the free stream of a wind tunnel indicated the static-pressure probe was insensitive for angles of yaw (to the mean flow) of approximately 10°.

The mean flow direction above the plate surface was determined by a constant-temperature hot-wire anemometer. The hot-wire probe was rotated in the (r, y)-plane and the location of maximum heat transfer determined. Because of the extremely high turbulence level of the wall jet flow it was necessary to use long time integration techniques (Chao 1965) to determine the statistical direction of the mean flow. Beyond a distance of 4·5 in. out from the surface it was extremely difficult to obtain a reliable indication of the statistical flow direction.

Mean turbulent velocity measurements were made using hot-wire anemometry techniques. The large turbulence levels made it difficult to use a Pitot-static probe for mean velocity determinations. Turbulence corrections of the order of 10 % were required to bring Pitot-static probe measurements into agreement with hot-wire results. Correction of the Pitot-static pressures by a Δq due to the turbulent velocities gave fair agreement with the hot-wire measurements.

A constant-temperature hot-wire anemometer of the type devised by Kovasznay (Kovasznay, Miller & Vasudeva 1963) was employed for the major part of the measurements. Where possible, direct graphic evaluation of the turbulent fluctuations was made. X-wire probes were employed to evaluate the turbulent velocities normal to the mean flow, as well as the turbulent shear stress, and it was necessary to make certain linear assumptions to evaluate the data thus obtained. In each case the X-probe hot-wire was calibrated as a function both of velocity and of yaw. For the low velocities the hot-wire was calibrated in a rotating arm-tank. Calibration of the wires was checked for every period before and after the data were taken. The hot-wires used were 0·00012 in. in diameter and roughly 0·06 in. long. The error in neglecting the large fluctuation component v in the y-direction in the evaluation of the component u in the r-direction was checked by operating the hot-wire normal to the flow in both the (r, y)- and (r, θ)-planes. It was found that the v-component did not affect the u-component measurement within the readability of the measurements. Evaluation of hot-wire anemometer data required the neglect of all but the first-order term in turbulent intensities, see Chao (1965). Estimates of the deviation from linearity for an estimate of the neglect of higher-order turbulent intensity terms indicates that errors of about 10 % might be made. The error due to the effect of non-linearity is probably less than the uncertainty introduced in the evaluation of the X-wire data. The repeatability of the measurements was also checked, and it was found that it was always within the scatter of the data.

In order to evaluate the r-direction derivatives, measurements were made at the three locations close together (28, 30 and 32 in. from the centre of the impinging jet). Derivatives in the r-direction were in all cases estimates of the best linear fit to the three points.

3. Discussion of results

The time-averaged equations of motion for an axisymmetric flow are, in the radial direction,

$$U\frac{\partial U}{\partial r}+V\frac{\partial U}{\partial y}+\frac{\partial \overline{u^2}}{\partial r}+\frac{1}{r}(\overline{u^2}-\overline{w^2})+\frac{\partial \overline{uv}}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left(\frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}-\frac{U}{r^2}+\frac{\partial^2 U}{\partial y^2}\right), \tag{1}$$

and, in the vertical direction,

$$U\frac{\partial V}{\partial r}+V\frac{\partial V}{\partial y}+\frac{\partial \overline{uv}}{\partial r}+\frac{\overline{uv}}{r}+\frac{\partial \overline{v^2}}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 V}{\partial r^2}+\frac{1}{r}\frac{\partial V}{\partial r}+\frac{\partial^2 V}{\partial y^2}\right). \tag{2}$$

The equation of conservation of mass is

$$\frac{\partial U}{\partial r}+\frac{U}{r}+\frac{\partial V}{\partial y}=0, \tag{3}$$

where U, V, W are the mean velocity components and u, v, w are the fluctuation velocity components in the r-, y- and θ-directions, respectively.

It is not immediately obvious which terms can be neglected in the above equations. Near the surface the wall jet is similar to a boundary layer, so that only the vertical derivative will be of major importance in the viscous terms. For a

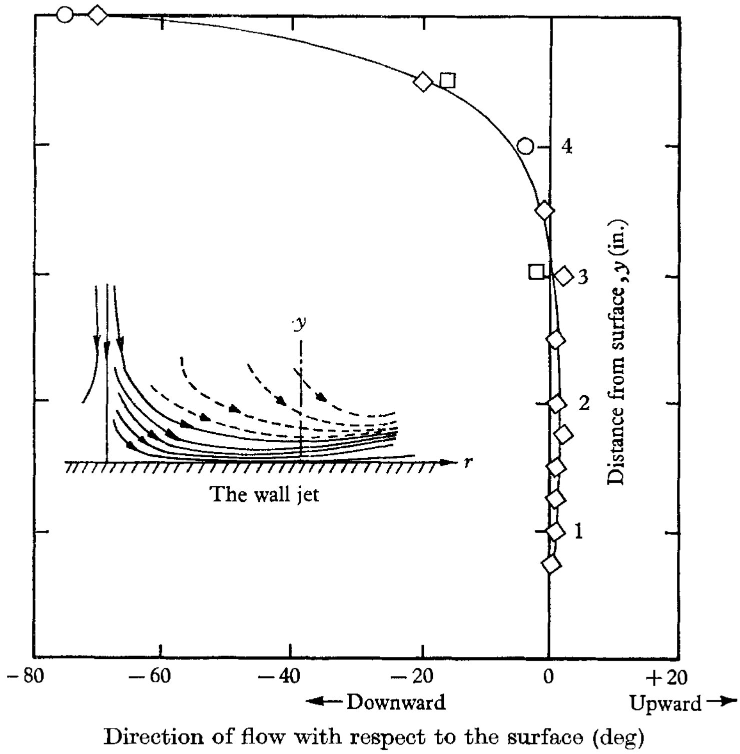

FIGURE 2. Measured mean flow direction above the surface. Distance from centre of vertical jet: □, r = 28 in. ◇, r = 30 in. ○, r = 32 in.

turbulent boundary layer it is possible to neglect the term $\partial\overline{u^2}/\partial r$ (Sandborn & Slogar 1955); however, there is no assurance that it can be neglected for the wall jet. Also, for the turbulent boundary layer the variation of pressure in the vertical direction does not affect the radial equation of motion. It is necessary, therefore, to measure the variation of pressure in the vertical and radial directions to determine if the two momentum equations are independent. There is no justification from order-of-magnitude arguments to eliminate the vertical equation of motion even for a turbulent boundary layer (Sandborn & Slogar 1955), so it is expected that the vertical equation is important in the wall-jet flow.

The purpose of the experimental programme was to determine the magnitude of the terms in (1) and (3). The actual measured points were plotted and a faired curve was drawn through the data in a way that appeared most consistent with the equations. In this way the measurements and the terms in the equations were employed as a cross-check on each other. Since the terms in the equations require differentiation of experimental data, they are less accurate than the prime data.

Figure 2 shows the variation of mean flow deflexion from a direction parallel to the surface. Positive angles indicate the flow deflected upward from the surface and negative angles indicate a downward flow. The air flow within the first 3 in. out from the surface is mainly supplied directly from the jet. Beyond about 3 in. out the flow is entraining the air surrounding the jet. Figure 2 contains an inset

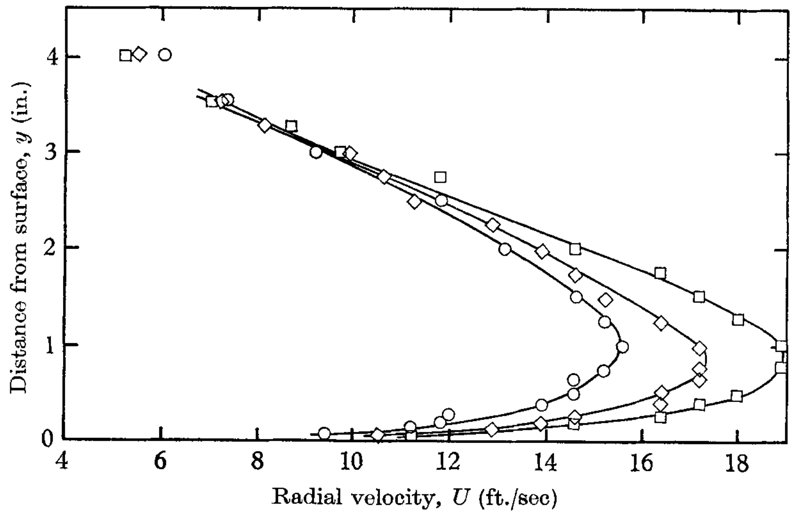

FIGURE 3. Mean radial velocity distributions. Distance from centre of vertical jet: □, r = 28 in. ◇, r = 30 in. ○, r = 32 in.

showing roughly the pattern of air flow. This flow pattern could be seen quite clearly by introducing very light particles (seed carriers from a cottonwood tree) into the outer entrained flow. These particles might make several revolutions in this outer vortex before they are swept out by the jet flow.

Figures 3 and 4 show the measured mean velocity distributions in the radial and vertical directions. Measurements further than 4 in. from the surface were not attempted because of the extreme variation in flow direction. The radial-velocity distributions were shown by Chao (1965) to agree with the similarity curve (α = 1·3) of Glauert (1956) over all but the extreme outer region of the flow. The measurements shown in figures 3 and 4 are sufficient to balance the equation of conservation of mass (3). Figure 5 shows the balance of the equation of conservation of mass. The uncertainty in the evaluation is noted as the shaded area on figure 5. The variation of the mean velocities with radial distance in the outer portion (greater than 2·5 to 3 in. from the surface) is so small that the present experimental accuracy is inadequate to determine it. Thus, the present evaluation is limited to the inner portion of the layer only.

The evaluation of the r-direction momentum equation requires the measurement of: the static pressure gradient in the radial direction; the turbulent-velocity, $\overline{u^2}$, gradient in the radial direction; the difference of the turbulent velocities,

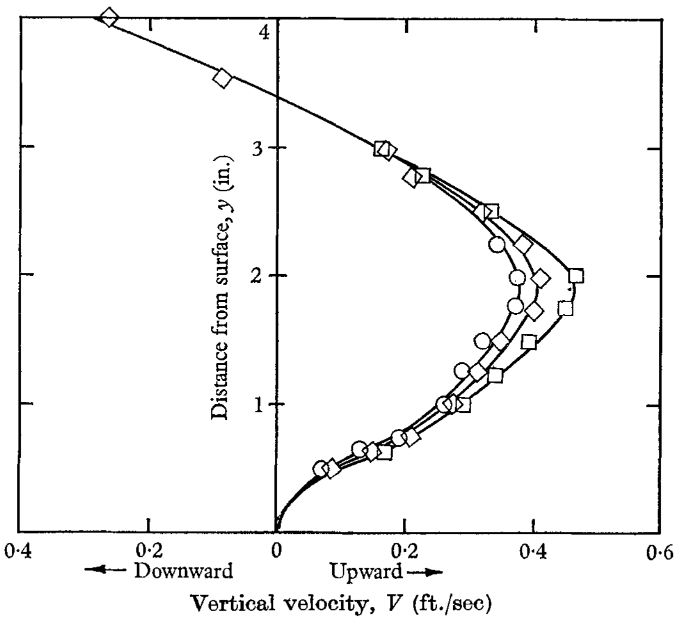

FIGURE 4. Mean vertical velocity distribution. Distance from centre of vertical jet: □, r = 28 in. ◇, r = 30 in. ○, r = 32 in.

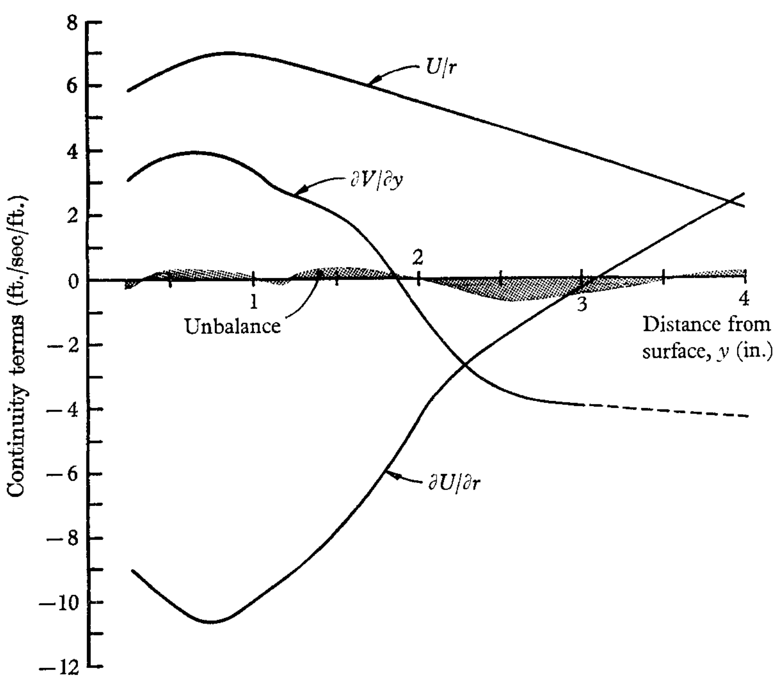

FIGURE 5. Terms appearing in the wall-jet conservation of mass equation.

$(\overline{u^2}-\overline{w^2})$; the vertical gradient of the turbulent shear term $\overline{uv}$; and also the mean-velocity terms.

Figures 6, 7 and 8 are plots of the data required to evaluate the r-direction momentum terms. Figure 6 is a plot of the static-pressure distribution across the layer for the three measuring stations. As may be seen from figure 6, the value of

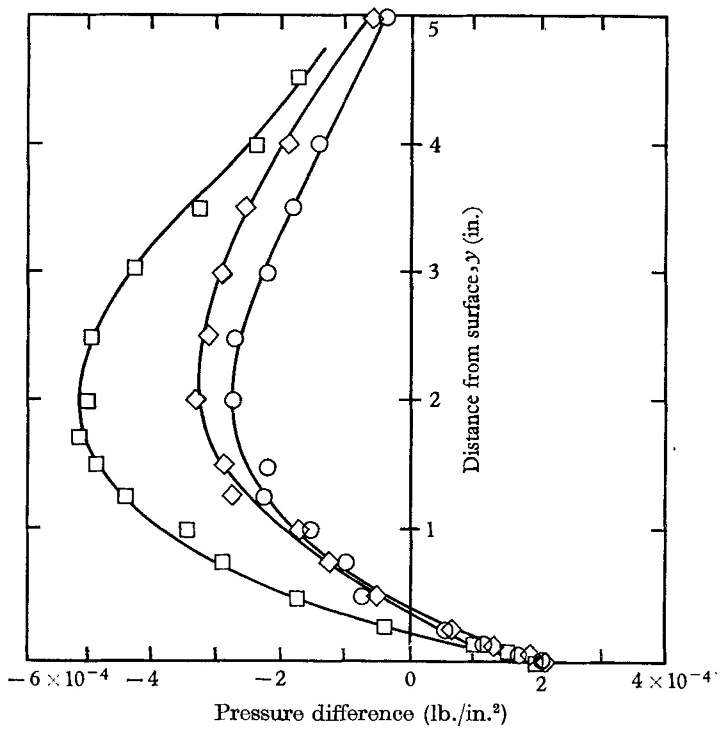

FIGURE 6. Static-pressure distribution across the wall jet. Distance from vertical jet: □, r = 28 in. ◇, r = 30 in. ○, r = 32 in. Reference pressure at top of sphere located at r = 24·37 in.

$\partial p/\partial r$ varies with vertical height. This variation of the pressure gradient through the shear region is markedly different from that for the boundary-layer-type flows. The variation of the pressure gradient also indicates that it will be impossible to neglect the vertical, y-direction equation of motion in an analytical evaluation of the flow. (Note that there is also a vertical pressure gradient around the surface of the sphere which was reported on in detail by Chao & Sandborn 1965.)

Figure 7 shows the measured values of the turbulent velocities. Only the radial component $(\overline{u^2})^{\frac{1}{2}}$ was evaluated for all three measuring stations. The inset on figure 7 (a) demonstrates the evaluation of $(\overline{u^2})^{\frac{1}{2}}$ from a hot-wire in both the vertical and horizontal positions. This check was deemed necessary, since the magnitude of $(\overline{v^2})^{\frac{1}{2}}$ was so large that it was felt that the linear assumptions made in evaluating the hot-wire signal were questionable. The normal turbulent velocity components, $(\overline{v^2})^{\frac{1}{2}}$ and $(\overline{w^2})^{\frac{1}{2}}$, were evaluated from X-wire measurements. Consistency checks show that the X-wire technique for evaluating turbulence is found to be accurate to approximately $\pm 20\,\%$ (Plate & Sandborn 1964).

Figure 8 is a plot of the turbulent shear stress, $\overline{uv}$, evaluated at the $r = 30$ in. station. The value of 'drag' measured on the sphere at the surface (Chao & Sandborn 1965) would occur at $-6\,\text{ft.}^2/\text{sec}^2$ on figure 8. There is some question as to the shape of the turbulent-shear-stress curve at the surface because of the presence of the spheres. Unfortunately, it was not forseen at the time of the

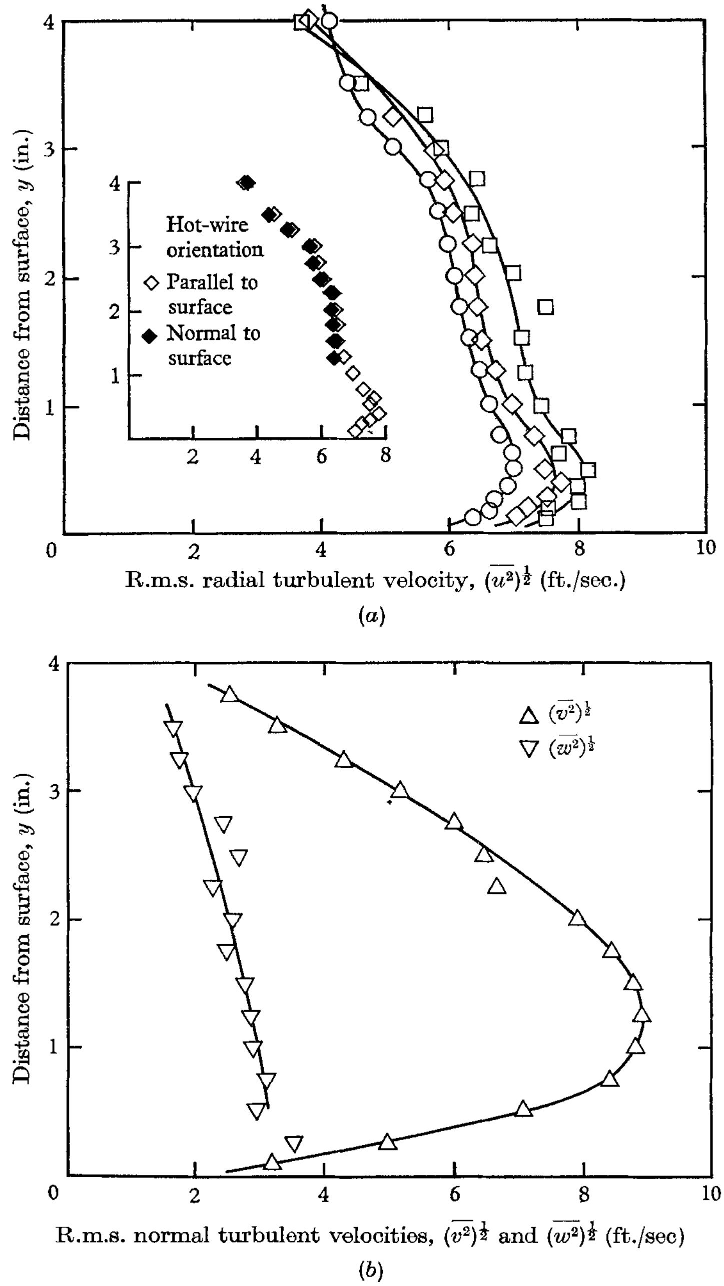

FIGURE 7. Turbulent velocities. (a) Distance from centre of vertical jet: □, $r = 28$ in. ◇, $r = 30$ in. ○, $r = 32$ in. (b) Normal turbulent velocities. $r = 30$ in.

measurements, but it now appears that $(\partial\overline{uv}/\partial r)$ is of importance in the vertical-direction equation of motion.

Figure 9 shows the evaluation of terms in the equation of motion, where the shaded portion represents the experimental error in balancing the equation. Of the terms, only $V(\partial U/\partial y)$ is smaller than the uncertainty. Beyond 2 in. from the

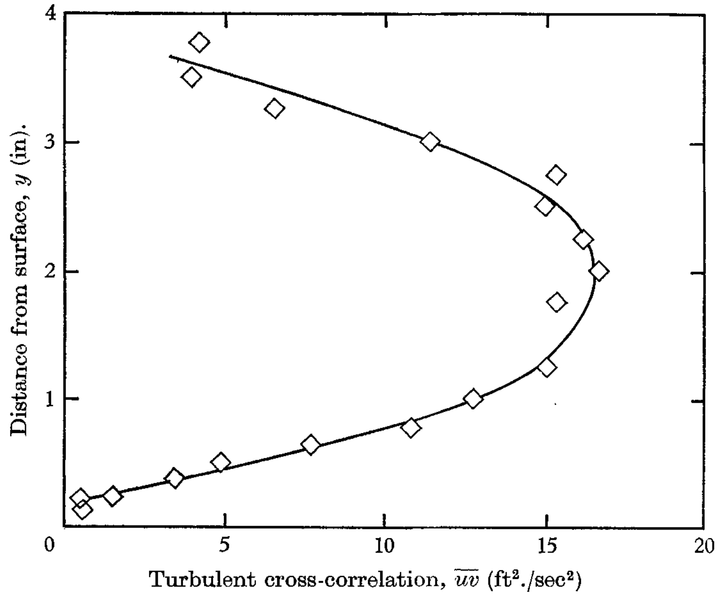

FIGURE 8. Distribution of the turbulent cross-correlation. $r = 30$ in.

surface (which is the height at which the velocity maximum occurs) the terms all become small and the experimental uncertainty does not justify further extension of the balance. If the term $V(\partial U/\partial y)$ were neglected as being of the order of the experimental uncertainty, then the unbalance is greatly reduced. This suggests that the radial-direction equation of motion for a wall jet may be reduced to the following:

$$U\frac{\partial U}{\partial r}+\frac{\partial \overline{u^2}}{\partial r}+\frac{1}{r}(\overline{u^2}-\overline{w^2})+\frac{\partial \overline{uv}}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\frac{\partial^2 U}{\partial y^2}. \tag{4}$$

The viscous term $\nu(\partial^2 U/\partial y^2)$ is not shown on figure 9, since it appears only very near the wall, where measurements of such terms as $\overline{uv}$ are not available.

At the outset it was assumed that the vertical-direction equation of motion would be a balance between the static pressure and the vertical turbulent velocity $(\overline{v^2})^{\frac{1}{2}}$. Such is not the case for the present set of data. Attempts to check the measured values of $(\overline{v^2})^{\frac{1}{2}}$ have not shown any great reduction in the values of $(\overline{v^2})^{\frac{1}{2}}$ reported in figure 7 (*b*). Further, since these values of $(\overline{v^2})^{\frac{1}{2}}$ were obtained at the same time that $\overline{uv}$ was evaluated it is felt that there is no major inconsistency in their values. The logical conclusion is that the vertical equation is more complicated than the simple pressure–turbulent velocity relation obtained for boundary layers (Sandborn & Slogar 1955). Of the terms in the vertical equation (2), the term $\partial\overline{uv}/\partial r$ appears to be the only one that might be of sufficient magnitude to balance the excess vertical turbulence term.

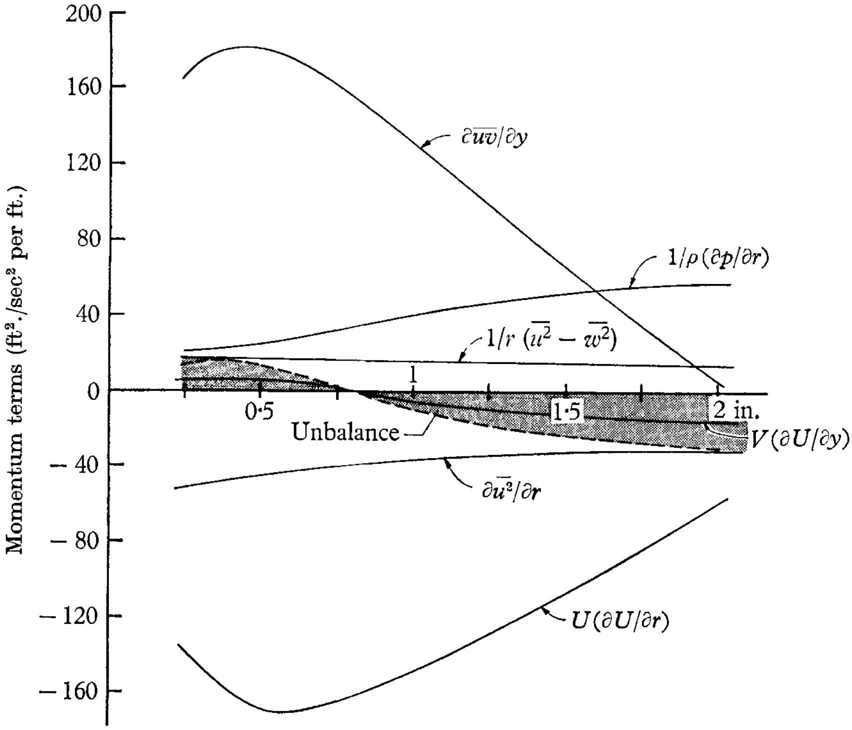

FIGURE 9. Wall-jet equation of motion.

4. Conclusions

The present experimental evaluation of the wall-jet equation of motion shows it to be more complicated than that for most shear flows. The existence of large vertical pressure effects is somewhat unique to this type of shear flow. The presence of the uniformly rough surface is thought to increase the magnitude of the turbulence in the flow; however, no attempt was made to compare the results with a smooth surface. The rough surface should be closely related to the applications encountered in nature.

Aside from the fact that the radial equation of motion is complicated for the wall jet, there is evidence that the vertical equation of motion must also be considered in any analytical treatment. Extension of the present work must include a definite attempt to fully evaluate the vertical equation.

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